Communications to the Editor

Comment on "Thermodynamics of the Separation of Biomaterials in Two-Phase Aqueous Polymer Systems: Effect of the Phase-Forming Polymers"

Recently Baskir et al.[1] published an adaptation of Scheutjens and Fleer's theory[2] for adsorption of a polymer from solution onto flat surfaces to polymer adsorption onto spherical particles. The primary goal of their study was to gain insight into the interaction between globular proteins and polymers in (semidilute) solutions. Although this study is of considerable interest, some of the new equations presented are in our opinion needlessly approximate and from a fundamental standpoint even incorrect. We will locate the error and suggest a consistent set of equations. A similar mistake was made in a paper of Ben-Shaul et al.[3] Our suggestion is also of value for the modeling of spherical aggregates, i.e., surfactant micelles[4] or lipid vesicles.[5]

The theory of Scheutjens and Fleer is a liquid lattice theory; i.e., real space is modeled by a lattice where the lattice sites are organized in layers and where each lattice site is either occupied by a solvent molecule or a polymer segment. Concentration gradients are only retained in one direction because of a mean-field approximation in the two other directions. That is why the lattice geometry determines the geometry of the system which can be modeled. Adsorption onto globular particles clearly requires the use of lattices with spherical symmetry.

Such a lattice consists of concentric shells or "layers", each containing a different number of lattice sites. The layers are indexed by a layer number, i, starting with $i = 1$ at the surface of the particle or, in the absence of a particle, at the innermost layer. Each layer has $L(i)$ sites, where in general $L(i) \neq L(j)$ for $i \neq j$. A lattice is characterized by its coordination number Z and the local geometry by the fraction $\lambda_i(j - i)$ of neighboring sites a site of layer i has in layer j. Obtaining expressions for the λ's is troublesome in the sense that curved lattices are, to a certain extent, artificial, and physical choices have to be made regarding the architecture of such a lattice. In spite of this, $\lambda_i(j - i)$ does play an important role in the calculation of the partition function of the polymer–solvent system. A wrong choice of $\lambda_i(j - i)$ as a function of i may seriously damage the validity of the statistics based on the lattice in general.

To illustrate our point, let us calculate the degeneracy of a ghost chain, i.e., a polymer chain allowed to self-intersect, of given order or "configuration" on an empty lattice. From this quantity the degeneracy of the entire system can be obtained, by using suitable correction factors, and from that the configurational entropy. These correction factors do not involve the use of λ's so we will not have to take these into account in the discussion. In the framework of the basic model of Scheutjens and Fleer, a conformation of a chain of r segments is defined by the respective layer numbers where the consecutive segments can be found. Starting to index the segments at one end of the chain, $s = 1, 2, ..., r$, and defining the layer number in which we can find segment s of conformation c as $i_{s,c}$, the number of ways ω_c of placing a chain in conformation c reads

$$\omega_c = L(i_{1,c})Z^{r-1}\prod_{s=1}^{r-1}\lambda_{i_{s,c}}(i_{s+1,c} - i_{s,c}) \qquad (1)$$

Note that here ω_c has a slightly different, but more convenient, definition as in the paper of Scheutjens and Fleer.[2] It is clear from expression 1 that in the calculation of ω_c we started counting at the very first segment $s = 1$. It needs no explanation that ω_c should be invariant to where, i.e., at which segment along the chain, the counting is started. For instance if we start counting at the other end of the molecule, at segment $s = r$, the result $\omega_{c,\text{inv}}$, would be

$$\omega_{c,\text{inv}} = L(i_{r,c})Z^{r-1}\prod_{s=r-1}^{1}\lambda_{i_{s+1,c}}(i_{s,c} - i_{s+1,c}) \qquad (2)$$

Invariance of the counting procedure requires that ω_c equals $\omega_{c,\text{inv}}$ or, and we would rather use this criterion, that

$$\omega_c/\omega_{c,\text{inv}} = 1 \qquad (3)$$

Since (3) contains only lattice parameters, the validity of an expression of the configurational entropy depends decisively on the internal consistence of the expressions for $L(i)$ and $\lambda_i(j - i)$. One can show that for the lattice used by Baskir et al. the following relation holds

$$\frac{\lambda_i(k - i)}{\lambda_k(i - k)} = \left(\frac{L(k)}{L(i)}\right)^2 \quad \text{for all} \quad |i - k| \leqslant 1 \qquad (4)$$

Equation 4 is useful in calculating the ratio $\omega_c/\omega_{c,\text{inv}}$, which becomes

$$\omega_c/\omega_{c,\text{inv}} = L(i_{r,c})/L(i_{1,c}) \neq 1 \qquad (5)$$

for all $i_r \neq i_1$. We see that for the λ's used by Baskir et al. the so-called inversion symmetry is not obeyed. They noticed this fact, however, and suggest that ω_c be approximated by

$$\omega_c = [\prod_{s=1}^{r}L(i_{s,c})^{1/r}]Z^{r-1}\prod_{s=1}^{r-1}[\lambda_{i_{s,c}}(i_{s+1,c} - i_{s,c})\lambda_{i_{s+1,c}}(i_{s,c} - i_{s+1,c})]^{1/2} \qquad (6)$$

thereby assuring inversion symmetry. The authors do not seem to realize that introduction of (geometrical) averages in an expression for ω_c causes an a priori loss of effects of curvature in the statistics. For if $\lambda(1)$ is the limiting value of $\lambda_i(+1)$ and $\lambda_i(-1)$ for large i, one would on geometrical grounds expect that $\lambda_i(+1) \geqslant \lambda(1) \geqslant \lambda_{i+1}(-1)$, and a geometrical averaging might very well neutralize any dependence on the layer number of ω_c. Indeed, using eq 5 of Baskir et al., the layer number dependence, i.e., curvature, disappears almost entirely. Having observed this fact, the authors even decided to keep the λ's constant throughout the lattice in the derivation of the partition function and density profiles, except for the calculation of the internal energy, leading to an inconsistency: the number of segment–solvent contacts does not equal the number of solvent–segment contacts any more. (Equation 14 of the author's article is not obeyed.)

As an alternative, we base the architecture of our spherically symmetric lattice on the following assumptions: (1) all lattice cells have equal volume; (2) the lattice layers are equidistant; (3) the number of contacts of layer i with layer j should be equal to the number of contacts layer j has in layer i, i.e.,

$$ZL(i)\lambda_i(j - i) = ZL(j)\lambda_j(i - j) \qquad (7)$$

(4) the number of neighbors a site or cell has in a layer is

0024-9297/88/2221-1876$01.50/0

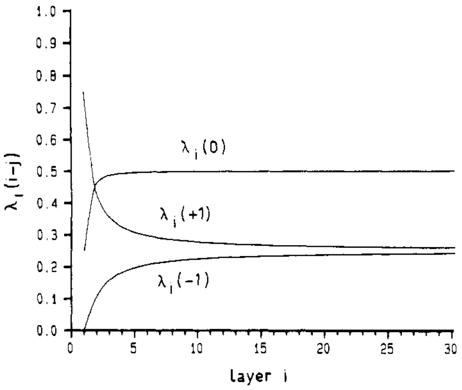

Figure 1. Fraction of neighbors of a site in layer i of a spherical lattice which are in the next layer, $\lambda_i(+1)$, in the same layer, $\lambda_i(0)$, and in the previous layer, $\lambda_i(-1)$, as a function of layer number V (hexagonal lattice).

proportional to the contact area this cell has with that layer, or

$$\lambda_i(+1)/\lambda_i(-1) = S_i/S_{i-1} \qquad \text{for all } i > 1 \tag{8}$$

where S_i stands for the contact area between layers i and $i + 1$. The number of lattice sites for layer i is given by the difference in volume of two spheres with a radius i and $i - 1$, respectively: $L(i) = V_i - V_{i-1}$. For a spherical lattice $V_i = {}^4/_3\pi i^3$ (in units of single cell volume) and the contact area between layers i and $i + 1$ is given by $S_i = 4\pi i^2$. The final result is

$$\lambda_i(+1) = \lambda(1)\frac{S_i}{L_i} \qquad \lambda_i(-1) = \lambda(1)\frac{S_{i-1}}{L_i}$$
$$\lambda_i(0) = 1 - \lambda_i(+1) - \lambda_i(-1) \tag{9}$$

for $i > 0$. The inversion symmetry requirement (eq 3) is met, as one can easily verify by using eq 7: in fact ω_c becomes totally invariant to the method of counting. It appears that according to our equations $\lambda_i(+1)$ and $\lambda_i(-1)$ are significantly different from their counterparts in a flat lattice, even for relatively large values of i (see Figure 1).

In conclusion, the model of Scheutjens and Fleer can be adapted to spherically symmetric systems in a consistent way by applying eq 9.

Acknowledgment. We thank Jan Scheutjens for helpful discussions.

References and Notes

(1) Baskir, J. N.; Hatton, T. A.; Suter, U. W. *Macromolecules* 1987, *20*, 1300.
(2) Scheutjens, J. M. H. M.; Fleer, G. J. *J. Phys. Chem.* 1979, *83*, 1619.
(3) Ben-Shaul, A.; Szleifer, I.; Gelbart, W. M. *Proc. Natl. Acad. Sci. U.S.A.* 1984, *81*, 4601.
(4) Leermakers, F. A. M.; van der Schoot, P. P. A. M.; Scheutjens, J. M. H. M.; Lyklema, J. In *Surfactants in Solution, Modern Applications*; Mittal, K. L., Ed.; Plenum: New York, in press.
(5) Leermakers, F. A. M.; Scheutjens, J. M. H. M., submitted for publication in *J. Chem. Phys.*

* To whom correspondence should be addressed.

† Present address: Department of Chemical Engineering and Materials Sciences, Laboratory for Technology of Macromolecular Compounds, Delft University of Technology, Julianalaan 136, 2628 BL Delft, The Netherlands.

P. P. A. M. van der Schoot† and F. A. M. Leermakers*

Department of Physical and Colloid Chemistry
Agricultural University Wageningen
De Dreijen 6, 6703 BC Wageningen, The Netherlands

Received December 2, 1987

Reply to Comment on "Thermodynamics of the Separation of Biomaterials in Two-Phase Aqueous Polymer Systems: Effect of the Phase-Forming Polymers"

In order to investigate the behavior of biomaterials, specifically of proteins, in two-phase polymer solutions we have recently modified[1] Scheutjens and Fleer's liquid lattice model[2,3] for the adsorption of polymer chains to a flat surface. Since particles usually have nonplanar surfaces the adaptation consisted of recasting Scheutjens and Fleer's model in a lattice of given (usually variable) curvature around a curved surface. At the heart of such an endeavor is the choice of a functional form for the directional weighting factors, $\lambda_i(j - i)$, which are the geometrical weighting factors for placement of the next chain segment in layer j given that the present one is in layer i. We chose the physically intuitive form of the ratio between the number of sites in layer j to that in layer i. van der Schoot and Leermakers[4] now take issue with our treatment of curvature in this liquid lattice model; they have chosen[5] a different form for λ_i, specifically constructed to avoid the inconsistencies to which the earlier choice leads.

van der Schoot and Leermakers are correct when they note that the effect of the curvature of the lattice on the combinatorial factor (which describes the number of different ways of arranging the polymer chains and solvent molecules in the lattice) is attenuated by our approximating the directional weighting factors, λ_i, by a local geometric average (to ensure "inversion symmetry") and that in our treatment the computed number of segment–solvent contacts is only approximately equal to the number of solvent–segment contacts (i.e., that our[1] eq 14 does not strictly hold). van der Schoot and Leermakers point[4] to an alternate definition of λ_i, given by Leermakers, van der Schoot, Scheutjens, and Lyklema;[5] this new form (eq 9 in their paper) indeed satisfies the condition that the number of segment–solvent contacts must be equal to the number of solvent–segment contacts. Values for λ_i computed with their formulas are closer to those obtained for a planar lattice (i.e., the limiting values as $i \to \infty$) than those computed[6] with our[1] eq 5 by roughly a factor of 2 (for these comparisons, we set the radius of the spherical particle, R, to zero), and since the λ_i's enter not only into the calculations of the combinatorial factor but also into those of the system energy, a compensation takes place: the new model[4,5] is more sensitive to changing curvature than ours[1] in the combinatorial factor, but less so in the energy. Also, we find the appeal of this new definition for λ_i, which is based on a ratio of the surface area to the volume of the layer, somewhat tempered by its dimensional inconsistency.

To assess the effect that changing our definition of λ_i to that of Leermakers et al.[5] would have on the results reported in our paper,[1] we have recalculated most values reported earlier. It is comforting to be able to state that the changes in the calculated Gibbs energies are minimal. A plot of the Gibbs energy per particle as a function of the polymer–particle interaction parameter χ_s, computed with Leermakers et al.'s[5] as well as with our definitions for λ_i, is shown in Figure 1 (the case shown is identical with the one depicted in Figure 5 of the original paper[1]). Also, the slight influence of the change of formulas used for λ_i on the Gibbs energy is systematic, so that the values of the distribution coefficients between different phases are not perceptibly different. Since the results published earlier are not changed significantly, we are led to believe that the particle surface area and the volume of lattice sites in a layer, which are calculated identically in the two model